of the temperature of thermal expansion of the product [4], which also results in diminution of the process intensity.

Therefore, questions of the stability of the isometric interfacial phase surfaces should certainly be taken into account in the construction of sublimating installations and the determination of optimal modes for their operation.

NOTATION

λ, heat-conduction coefficient; ρ, density; L, specific heat of sublimation; x, y, h, coordinates; R, H, tablet radius and thickness; q, heat flux; σ, rms deviation; a, thermal diffusivity coefficient; τ_f, time for fluctuation development; τ_{pl}, time of plane front passage; T_r, temperature of the ice surface at the side of the heated substrate; T_e, equilibrium temperature on the sublimation boundary.

LITERATURE CITED

1. A. Z. Volynets, Inzh.-Fiz. Zh., 15, No. 1, 162-164 (1968).
2. Ya. Ber, D. Zaslavski, and S. Irmay, Physicomathematical Principles of Water Filtration [in Russian], Moscow (1971).
3. V. I. Lavrik and V. N. Savenkov, Handbook on Conformal Mappings [in Russian], Kiev (1970).
4. A. M. Brazhnikov, E. I. Kaukhcheshvili, and A. I. Vasil'ev, Technique and Technology of Sublimation Drying of Products, Trudy, KTIRPiKh, No. 69 38-41 (1973).

MELTING OF PORE ICE WITH THE FORMATION OF AN EXTENDED ISOTHERMAL ZONE

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The melting of pore ice, characterized by the formation of a transition zone in which the two phases simultaneously coexist, is investigated. The extent of the transition zone is related to the rate of inflow of water into the melted region of the pore space from outside.

Usually, the melting of ice in the pores of a coarse-grained medium is described mathematically in terms of the classical Stefan problem, on the assumption that the regions of different states of aggregation of the water are separated by a surface of zero thickness. This assumption does not always have to be made and, as shown in [1, 2], even in a homogeneous body it is possible for the front to split and form an extended isothermal zone. Obviously, in composite media, e.g., in frozen sand, in which ice is one of the components, there are more conditions that determine the splitting of the front. One of these is a higher rate of heat transfer in the mineral skeleton than in the pore ice. As a result, there is formed a zone of coexistence of water and ice in which the water coats the warm particles of the skeleton while the ice occupies the centers of the pores.

The solution of the model problem of the melting of sheets of ice alternating with sheets of quartz of equal thickness has shown that this zone is longest at the beginning of the process and subsequently contracts to a small but finite length. This result was obtained on the assumption that no water enters the melting zone from outside, and it is in this case that the formation of a front after a fairly long interval is observed.

The inflow of water from the outside can retard the contraction of the transition zone and ultimately lead to the splitting of the front. The inflow of water is the result of the specific volume of the ice decreasing by an amount $(\rho_w - \rho_i)/\rho_w$, which leads to a sharp decrease in the pressure in the pore space, if it is isolated from the external medium, to a value corresponding to the vapor tension of the water. Later, this observation will be used to estimate the pressure at the leading front.

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In order to investigate this process we will consider a homogeneous sand bed occupying the half-space ($x \geqslant 0$) whose pores are initially completely full of ice. We assume that at each moment of time the region of joint coexistence of water and ice occupies a prismatic volume bounded by the moving coordinatex $x_m(t)$ and $x_s(t)$, at which the constant temperatures $T_m = T_s = 0°C$ are given.

This idealization does not contradict the physical possibilities. As is known, under thermostatically controlled conditions at 0°C water and ice can coexist for an infinitely long time in dynamic equilibrium: if some small mass of ice melts, the same mass of water will freeze, and the fall in temperature associated with the thawing of the ice will be compensated by the increase associated with the freezing of the water. However, in the presence of a transition zone this equilibrium is disturbed because the thawing of a small mass of ice results in the formation of a free space which is quickly filled by warmer water from the adjacent, completely thawed out zone. Thus, under these conditions the thawing of the ice does not involve the freezing of water and the process develops in a particular direction.

By definition, in the region $0 \leqslant x \leqslant x_m$ the pore space of the bed is occupied by water, while in the region $x_s \leqslant x < \infty$ the pores are full of ice and this part is impermeable for the water. In the zone $x \in [x_m, x_s]$ water and ice simultaneously coexist. Here the permeability $k(\sigma)$ varies monotonically with the water saturation $\sigma(x, t)$ from zero at the leading front to the total permeability of the bed at the trailing front k_m, where the water saturation $\sigma(x_m, t)$ is taken equal to unity.

If the water is assumed to be incompressible, then, obviously, the variation of the mass of the water entering this zone will be determined by compensation of the volume deficit associated with the thawing of the ice, i.e.

$$-\frac{\partial v}{\partial x} = \Delta \frac{\partial \sigma}{\partial t}; \quad \Delta = m \frac{\rho_w - \rho_i}{\rho_w}. \tag{1}$$

Given the assumptions, at the leading front the flow ceases ($v(x_s, t) = 0$), and at the trailing front the percolation rate has the same value as at the entrance to the bed, i.e., $v(x_m, t) = v(0, t)$.

The integration of (1) gives

$$v(x_m, t) = \Delta \frac{d}{dt}\Big[\int_{x_m}^{x_s} \sigma dx + x_m\Big]. \tag{2}$$

The percolation rate is related to the pressure field in the completely and partially thawed zones by the equations [3]

$$\frac{\partial^2 p}{dx^2} = 0; \; x \in [0, x_m]; \quad \frac{\partial}{\partial x} \frac{k(\sigma)}{\mu} \frac{\partial p}{\partial x} = \Delta \frac{\partial \sigma}{\partial t}; \; x \in [x_m, x_s]. \tag{3}$$

From this there follow the equations

$$v(0, t) = v(x_m, t) = \frac{k_m}{\mu} \frac{p_0 - p_m}{x_m}; \tag{4}$$

$$p(x_m, t) - p(x_s, t) = \Delta\mu \int_{x_m}^{x_s} \frac{dx}{k(\sigma)} \int_{x}^{x_s} \frac{\partial \sigma}{\partial t} dx, \tag{5}$$

where $p_0 = p(0, t)$.

Since at the leading front the permeability tends to zero, as indicated above the melting of the ice behind it leads to the pressure falling to an insignificant value, so that we can take

$$p(x_s, t) = 0. \tag{6}$$

By assumption, at the trailing front the entire cross section of the bed is occupied by water, i.e. $\sigma(x_m, t) = 1$. At the leading front, on the other hand, the cross section is wholly occupied by ice, so that $\sigma(x_s, t)=0$. In each cross section the ice content of the transition zone per unit volume of the bed is $m(1-\sigma)$ and it varies from the trailing to the leading front as a function of the amount of heat supplied by the water. Through the trailing front energy equal to $v_m\rho_w L$ is supplied to the transition zone by convection, while the heat flux q_s passes through the leading front into the frozen medium. If heat losses through the roor and floor of the bed are neglected, the difference is wholly expended on melting the ice, i.e.

$$v_m\rho_w L - q_s = m\rho_i L \int_{x_m}^{x_s} \frac{\partial}{\partial t}(1-\sigma)\,dx.$$

On the other hand, the convective heat flux $v_m\rho_w L$ is determined by the amount q_m supplied in conductive form from the thawed region to the trailing front. As a result, the heat balance in the transition zone is given by the equation

$$q_m - q_s = m\rho_i L \frac{d}{dt}\left[\int_{x_m}^{x_s} \sigma dx + x_m\right]. \qquad (7)$$

In the absence of a transition zone, when $x_m = x_s$, Eq. (7) corresponds to the usual Stefan condition at a phase transition front. In the formulation in question the heat balance is related to the amount of water supplied to the bed, as may be seen from comparing (7) with (2).

For modeling purposes we assume that in this zone σ and $k(\sigma)$ vary linearly:

$$\sigma = \frac{x_s - x}{x_s - x_m};\; k(\sigma) = k_m \frac{x_s - x}{x_s - x_m}. \qquad (8)$$

Substituting these relations in (2) and (5), with allowance for (6), gives

$$v(0,\ t) = \frac{1}{2}\Delta(\dot{x}_m + \dot{x}_s);\quad p(x_m,\ t) = \frac{\Delta\mu}{2k_m}(x_s - x_m)(\dot{x}_s - \dot{x}_m). \qquad (9)$$

Using (4), we find the following relation between the coordinates of the leading and trailing fronts and their derivatives $\dot{x} = dx/dt$:

$$\frac{\Delta\mu}{2k_m}[(\dot{x}_s + \dot{x}_m)x_m + (x_s - x_m)(\dot{x}_s - \dot{x}_m)] = p_0. \qquad (10)$$

A second relation between the coordinates of the two fronts is obtained by combining (8) and (7) in the form:

$$q_m - q_s = \frac{1}{2} m\rho_i L(\dot{x}_m + \dot{x}_s). \qquad (11)$$

In the simplest case, when the pressure and temperature at the inlet surface ($x = 0$) are constant and equal to p_0 and T_0 respectively, and the initial temperature of the bed T_i is also constant, the problem admits the self-similar solution

$$x_m = 2a\sqrt{\alpha_1 t};\; x_s = 2b\sqrt{\alpha_1 t}, \qquad (12)$$

where α_1 is the thermal diffusivity of the thawed zone; a and b are certain constants determined in the course of solving the problem. In this case the temperature distributions within the thawed and frozen zones are given by the following expressions:

$$\frac{T_1 - T_m}{T_0 - T_m} = 1 - \operatorname{erf}\frac{x}{2\sqrt{\alpha_1 t}} \Big/ \operatorname{erf} a,$$

$$\frac{T_s - T_2}{T_s - T_i} = -1 + \operatorname{erfc}\frac{x}{2\sqrt{\alpha_2 t}} \Big/ \operatorname{erfc} b\sqrt{\beta},\; \beta = \frac{\alpha_1}{\alpha_2}.$$

Since $q_m = -\lambda_1 (\partial T_1/\partial x)_{x_m}$; $q_s = -\lambda_2 (\partial T_2/\partial x)_{x_s}$, from (11) and (10), using (12), we obtain

$$\frac{K_1}{\exp(-a)\operatorname{erf}\sqrt{a}} - \frac{\sqrt{\beta} K_2}{\exp(-\beta b)\operatorname{erfc}\sqrt{\beta b}} = \frac{\sqrt{\pi}}{2}(a+b), \tag{13}$$

and from (10), together with (13),

$$M = a(a+b) + (b-a)^2, \tag{14}$$

where

$$M = \frac{p_0}{\alpha_1 \Delta}\frac{\mu}{k_m};\; K_1 = \frac{\lambda_1 (T_0 - T_m)}{\alpha_1 m \rho_i L};\; K_2 = \frac{\lambda_2 (T_s - T_i)}{\alpha_1 m \rho_i L}. \tag{14}$$

When b = a, and the leading and trailing fronts coincide, Eq. (13) gives the classical Stefan solution, denoted below by a_0, while (14) is transformed into the expression $M_0 = 2a_0^2$, which gives the pressure at the inlet surface of the bed determined by the suction. If the pressure at the inlet surface increases, i.e., if it is assumed that $M > M_0$, then Eqs. (13) and (14) will determine the parameters of motion of the two fronts. In this case the relations $a<a_0$ and $b>b_0$ will always hold, i.e., the trailing front will lag behind the front corresponding to the classical Stefan solution, whereas the leading front will precede it. In Fig. 1 we have presented in graphic form the results of solving the problem for $K_1 = 0.1$, $K_2 = 0.5$, and $\beta = 1$.

As may be seen from Fig. 1, splitting of the pore ice thawing front is observed when $M > M_0 = 2a_0^2$, and the lower the value of a_0 the smaller the pressures at the inlet surface at which splitting of the front takes place. We note that, other things being equal, a_0 decreases monotonically at the same time as the temperature T_0 in the inlet section.

The results obtained can be used to investigate the disaggregation of sand cemented only by ice. In the frozen state such sand is a monolithic rock; when heat is supplied, the cementing effect of the ice is lost and the sand disintegrates. In the fully thawed out zone the originally monolithic sand may crumble into individual grains and in the transition zone into larger aggregations within which the cementing ice bonds remain intact. Obviously, if the heat supply remains the same, the disaggregation volume will be much greater for a split front than for ordinary thawing.

The disaggregation of sand cemented only by ice leads to the formation of voids during well drilling with muds at a positive temperature [4, 5]. As follows from the results obtained, in order to reduce these voids it is necessary not only to lower the temperature of the drilling mud but, above all, to prevent its aqueous filtrate from entering the bed.

Experiments were performed to check the qualitative agreement between the thawing model proposed and actual data. The frozen soil being drilled was simulated by a sand cylinder chilled to −2°C, through whose central cavity water at 2-3°C was pumped. In one series of experiments the water flow was separated from the frozen sand by a copper tube, and in a second by a fine mesh wrapped into a cylinder. Thus, in the first series we simulated the thawing of pore ice by a conductive heat flux only, while in the second the conductive heat flux was supplemented by infiltration of the water. Temperature sensors were installed over half the frozen layer and recorded the zero isotherm arrival time. Other things being equal, when a mesh cylinder was used the zero isotherm arrival time was reduced by a factor of 1.1-1.4 depending on the size of the sand grains; the greatest reducting being observed in the case of coarse-grained sands. Since for sandy media the permeability is basically determined by the grain size, the experimental data point to an acceleration of melting with increase in permeability, in full conformity with the ideas outlined above.

On the basis of the experiments and the theoretical analysis of their results, it has been proposed that to reduce the voids associated with well drilling in frozen soils using drilling muds, not only should the heat-transfer coefficient of the muds be reduced but the infiltration index (water loss) should also be lowered by adding special polymers: carboxymethyl cellulose, polyethylene oxide, etc. This recommendation has led to a reduction in sand evacuation from the wells by a factor of 1.3-1.6 and has made it possible to improve the quality of the borehole on the frozen-soil interval without lowering the mud temperature [5].

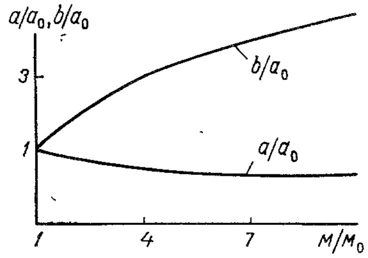

Fig. 1. Velocity ratios for the trailing a/a_0 and leading b/a_0 fronts bounding the extended isothermal zone relative to the classical Stefan solution as functions of the pressure at the inlet surface of the bed as given by the dimensionless parameter M/M_0.

In conclusion, we note that the proposed model of the front splitting process associated with the thawing of pore ice can be complicated by postulating nonzero water saturation at the leading front. In this case the pressure at the front is nonzero and is determined in the course of solving the problem; however, the qualitative picture remains the same and hence the conclusions concerning measures to reduce the disintegration of sand beds cemented only by ice during well drilling still hold good.

NOTATION

ρ, density; m is porosity; k, permeability; σ, water saturation; μ, viscosity; L, latent heat of fusion of the ice; x, distance; p, pressure; v, percolation velocity; T, temperature; t, time; q, heat flux; λ, thermal conductivity; α, thermal diffusivity; β, ratio of the thermal diffusivities in the thawed and frozen zones; Δ, K_1, K_2, M, a, and b, dimensionless parameters. Subscripts: w, i, water and ice; 1, 2, thawed and frozen zones; 0, inlet surface of the bed; i, initial temperature; and m and s, trailing and leading fronts.

LITERATURE CITED

1. A. M. Meirmanov, Dokl. Akad. Nauk SSSR, 258, No. 3, 547-549 (1981).
2. A. Fasano and M. Primicerio, "A parabolic-hyperbolic free boundary problem: mushy regions with variable temperature in melting processes," Preprint No. 4, Universita degli studi di Firenze, Instituto Matematico "Ulise Dini" (1982/1983).
3. A. E. Scheidegger, The Physics of Flow Through Porous Media, Macmillan, New York (1957).
4. R. I. Medvedskii and A. I. Kozubovskii, Problems of Tyumen' Oil and Gas [in Russian], No. 58 (1983), pp. 74-77.
5. R. I. Medvedskii, Problems of Tyumen' Oil and Gas [in Russian], No. 59 (1983), pp. 86-88.